

University of Groningen

## Supersymmetric R4-actions in ten dimensions

Roo, M. de; Suelmann, H.; Wiedemann, A.

*Published in:*  
Physics Letters B

*DOI:*  
[10.1016/0370-2693\(92\)90769-Z](https://doi.org/10.1016/0370-2693(92)90769-Z)

**IMPORTANT NOTE:** You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

*Document Version*  
Publisher's PDF, also known as Version of record

*Publication date:*  
1992

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Roo, M. D., Suelmann, H., & Wiedemann, A. (1992). Supersymmetric R4-actions in ten dimensions. *Physics Letters B*, 280(1), 39-46. [https://doi.org/10.1016/0370-2693\(92\)90769-Z](https://doi.org/10.1016/0370-2693(92)90769-Z)

**Copyright**

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

**Take-down policy**

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

*Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.*

# Supersymmetric $R^4$ -actions in ten dimensions

M. de Roo, H. Suelmann and A. Wiedemann

*Institute for Theoretical Physics, Nijenborg 4, NL-9747 AG Groningen, The Netherlands*

Received 20 January 1992

We construct supersymmetric  $R+R^4$ -actions in ten dimensions. Two invariants, of which the bosonic parts are known from string amplitude and sigma model calculations, are obtained. One of these invariants can be generalized to an  $R+F^2+F^4$ -invariant for supersymmetric Yang–Mills theory coupled to supergravity. Supersymmetry requires the presence of  $B \wedge R \wedge R \wedge R \wedge R$ -terms, ( $B \wedge F \wedge F \wedge F \wedge F$  for Yang–Mills) which correspond to counterterms in the Green–Schwarz anomaly cancellation.

## 1. Introduction

Ten-dimensional superstring effective actions, which are an important ingredient in phenomenological applications of superstring theories, are restricted by the requirement of local supersymmetry. The complete effective action consists of an infinite series of terms, with increasing powers of the Riemann tensor  $R$ . In this letter we present the supersymmetric completion of bosonic actions quartic in the Riemann tensor.

The following  $R^4$ -actions (in  $d=10$ ) have been discussed in this context:

$$X = t^{\mu\nu\lambda\rho\sigma\tau\alpha\beta} t_{abcde fgh} R_{\mu\nu}{}^{ab} R_{\lambda\rho}{}^{cd} R_{\sigma\tau}{}^{ef} R_{\alpha\beta}{}^{gh}. \quad (1.1)$$

This action was obtained from string amplitude considerations [1], and from a calculation of the two-loop  $\beta$ -function in a supersymmetric sigma-model [2]. It appears in the string effective action with the characteristic coefficient  $\zeta(3)$ . The tensor  $t$  is given explicitly in, e.g., ref. [3]<sup>#1</sup>. The action  $[t \cdots (\text{tr } R^2)^2]$

$$Y_1 = t^{\mu\nu\lambda\rho\sigma\tau\alpha\beta} R_{\mu\nu}{}^{ab} R_{\lambda\rho}{}^{ab} R_{\sigma\tau}{}^{cd} R_{\alpha\beta}{}^{cd} \quad (1.2)$$

we also found in tree-level string amplitude calculations [4]. One may also take a different trace over Lorentz indices, which leads to  $[t \cdots (\text{tr } R^4)]$

$$Y_2 = t^{\mu\nu\lambda\rho\sigma\tau\alpha\beta} R_{\mu\nu}{}^{ab} R_{\lambda\rho}{}^{bc} R_{\sigma\tau}{}^{cd} R_{\alpha\beta}{}^{da}. \quad (1.3)$$

A fourth action, which is by construction invariant under linearized supersymmetry transformations, is [5]

$$Z = R_{[ab}{}^{ab} R_{cd}{}^{cd} R_{ef}{}^{ef} R_{gh}{}^{gh}]. \quad (1.4)$$

If reduced to eight dimensions  $Z$  becomes a total derivative, and does not play a role in lightcone gauge string amplitude calculations. Therefore one has no a priori knowledge from string amplitudes about its presence in a ten-dimensional supersymmetric invariant. The fact that in ten dimensions one should allow the presence of  $Z$  was stressed in refs. [5–7].

Our main results are as follows. We find that the most general supersymmetric action of the type  $R + \gamma R^4$ , where  $\gamma$  is an arbitrary constant, is a linear combination of two actions which are separately invariant. The

<sup>#1</sup> In ref. [3] the indices on  $t$  take on only the eight transverse values. In this letter  $t$  indicates the tensor structure of ref. [3] without the eight-dimensional Levi-Civita symbol, but with indices taking all ten values.

bosonic part of the first invariant contains  $X$  and  $Z$ . The second invariant contains only  $Y_1$ . Note that the contractions of Lorentz indices in  $Y_1$  (and  $Y_2$ ) are as for an  $SO(9,1)$  Yang–Mills group, and indeed this invariant can be generalized to an arbitrary Yang–Mills group. In the abelian case this  $F^4$ -action coincides with the quartic terms of the Born–Infeld action [8]. As we shall see below, this requires the presence of the supersymmetric  $F^2$ -action.

The supersymmetric completion of these  $R^4$ -actions contains  $B \wedge R \wedge R \wedge R \wedge R$ -terms ( $B \wedge F \wedge F \wedge F \wedge F$  for Yang–Mills), where  $B$  is the two-index antisymmetric gauge field of  $d=10$ ,  $N=1$  supergravity. Such terms are known from the Green–Schwarz anomaly cancellation [9], where they appear as counterterms. Their presence in the low-energy effective action has also been established from the calculation of one-loop amplitudes for the heterotic string [10]. Therefore the term  $X$  in (1.1) can be considered as part of the supersymmetric completion of the Green–Schwarz counterterms.

The invariance of the  $R + \gamma R^4$ -action holds only iteratively in  $\gamma$ . Thus we allow changes, proportional to  $\gamma$ , in the supersymmetry transformation rules of the supergravity fields. The value of  $\gamma$  depends on the particular application of the result, and is of course not fixed by supersymmetry. The modifications of the transformation rules are relevant for compactification to four dimensions with unbroken  $N=1$  supersymmetry [11].

Results about the supersymmetrization of (1.1) have been obtained in superspace [12]. Since we do not want to limit ourselves to (1.1), we prefer the construction of the most general invariant containing  $R^4$  by the explicit Noether method, to the extraction of component results from ref. [12].

We emphasize that we do not consider the addition of an  $R^2$ -term in the action. Such terms occur in the supersymmetrization of Lorentz Chern–Simons terms [9]. Invariance holds iteratively in the coefficient of the  $R^2$ -term, the series also containing an  $R^4$ -term. The component version of this  $R^4$ -term, including Yang–Mills contributions, was worked out previously [13]. In section 4 we will discuss the relation of the present work with these results.

## 2. The construction

In  $N=1$ ,  $d=10$  supergravity [14] we have the following transformation rules under local supersymmetry:

$$\begin{aligned} \delta e_\mu^a &= \frac{1}{2} \bar{\epsilon} \Gamma^a \psi_\mu, \quad \delta \psi_\mu = (\partial_\mu - \frac{1}{4} \Omega_{\mu+}{}^{ab} \Gamma_{ab}) \epsilon, \quad \delta B_{\mu\nu} = \frac{1}{2} \sqrt{2} \bar{\epsilon} \Gamma_{[\mu} \psi_{\nu]}, \\ \delta \lambda &= -\frac{3}{8} \sqrt{2} \phi^{-1} \mathbb{D} \phi \epsilon + \frac{1}{8} \Gamma^{abc} \epsilon H_{abc}, \quad \phi^{-1} \delta \phi = -\frac{1}{3} \sqrt{2} \bar{\epsilon} \lambda. \end{aligned} \quad (2.1)$$

In this work we do not consider contributions to the action quartic in fermions, therefore there is no need to consider bilinear fermions in the above transformation rules<sup>#2</sup>. We have defined

$$\Omega_{\mu\pm}{}^{ab} = \omega_\mu{}^{ab}(e, \psi) \pm \frac{3}{2} \sqrt{2} H_\mu{}^{ab}. \quad (2.2)$$

From (2.1) and (2.2) one readily obtains

$$\delta \Omega_{\mu-}{}^{ab} = \frac{1}{2} \bar{\epsilon} \Gamma_\mu \psi^{ab}, \quad \delta \psi^{ab} = -\frac{1}{4} \Gamma^{cd} \epsilon R_{cd}{}^{ab}(\Omega_-), \quad (2.3)$$

where  $\psi_{ab}$  is the gravitino curvature:  $\psi_{\mu\nu} = \mathcal{D}_\mu(\Omega_+) \psi_\nu - \mathcal{D}_\nu(\Omega_+) \psi_\mu$ . Note that (2.3) coincides precisely with the transformation rules of the  $d=10$  Yang–Mills multiplet.

To make the supersymmetrization of  $R^4$ -actions feasible, a few further restrictions have to be made. We will only consider contributions to the action which are linear in the field strength  $H$  of the antisymmetric tensor gauge field  $B_{\mu\nu}$ . This implies that we do not consider variations of the action containing  $H$ . Furthermore, we do not consider terms in the action containing the Ricci-tensor or  $\Gamma^\nu \psi_{\mu\nu}$ , since these are proportional to equations of motion of the  $R$ -action, and can be eliminated by redefining the fields.

<sup>#2</sup> The complete form of the  $d=10$ ,  $N=1$  supergravity action and these transformation rules can be found in ref. [13].

The  $\gamma R^4$ -action itself will be invariant only up to terms proportional to the equations of motions, and therefore invariance of the  $R + \gamma R^4$ -action is achieved by modifying the supersymmetry transformation rules with terms of  $O(\gamma)$ . The determination of these modifications is one of the reasons for doing this explicit supersymmetrization. The modifications of the fermionic transformation rules will influence the compactification procedure from ten to four dimensions, as set out in ref. [11].

In this letter, we will consider only the sector which includes the zehnbein, the gravitino and the antisymmetric tensor gauge field  $B_{\mu\nu}$ . There are a number of contributions to the variation of the action which are insensitive of the presence of  $\lambda$  and  $\phi$ . As we shall see, the cancellation of these variations is already very restrictive. In this paper we only obtain the modifications of the gravitino transformation rule, the transformation rule of  $\lambda$  being left to a later publication.

The starting point is the construction of an ansatz for the action. Let us consider the  $R^4$ -terms explicitly. There are seven independent products of four Riemann tensors. A parametrization of this sector of the ansatz is given by

$$\begin{aligned} A_1 &\equiv R_{abef}R_{abef}R_{cdgh}R_{cdgh}, & A_2 &\equiv R_{acef}R_{bcef}R_{adgh}R_{bdgh}, & A_3 &\equiv R_{abef}R_{cdef}R_{abgh}R_{cdgh}, & A_4 &\equiv R_{abef}R_{cdef}R_{acgh}R_{bdgh}, \\ A_5 &\equiv R_{abef}R_{cdef}R_{agch}R_{bgdh}, & A_6 &\equiv R_{abef}R_{cdef}R_{agbh}R_{cgdh}, & A_7 &\equiv R_{abef}R_{cdef}R_{agch}R_{bgdh}. \end{aligned} \quad (2.4)$$

Schematically, the other sectors included in the present calculations are  ${}^{\#3} HR^2 \mathcal{D}R$ ,  $BR^4$ ,  $\bar{\psi}_{(2)}\Gamma\psi_{(2)}R \mathcal{D}R$ ,  $\bar{\psi}_{(2)}\Gamma \mathcal{D}\psi_{(2)}R^2$ ,  $\bar{\psi}\Gamma\psi_{(2)}R^3$  and  $\bar{\psi}\Gamma\psi R^2 \mathcal{D}R$ . The structure of most of these is obvious, except perhaps  $BR^4$ . These are two independent contributions:

$$K_1 \equiv i\epsilon^{\mu_1\dots\mu_{10}}B_{\mu_1\mu_2}R_{\mu_3\mu_4}{}^{ab}R_{\mu_5\mu_6}{}^{ab}R_{\mu_7\mu_8}{}^{cd}R_{\mu_9\mu_{10}}{}^{cd}, \quad K_2 \equiv i\epsilon^{\mu_1\dots\mu_{10}}B_{\mu_1\mu_2}R_{\mu_3\mu_4}{}^{ab}R_{\mu_5\mu_6}{}^{ac}R_{\mu_7\mu_8}{}^{bd}R_{\mu_9\mu_{10}}{}^{cd}. \quad (2.5)$$

They are clearly invariant under gauge transformations of the  $B$ -field,  $\delta B_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ . We find that both terms play a role in the invariants. Their properties will be further discussed in section 4. Altogether, there are approximately 200 terms, each with an a priori arbitrary coefficient, in the ansatz.

For later reference it is useful to rewrite  $X$ ,  $Y_1$ ,  $Y_2$  and  $Z$  (1.1)–(1.4) in terms of  $A_1$ – $A_7$ . The result is

$$\begin{aligned} X &= 12(A_1 - 16A_2 + 2A_3 - 32A_5 + 16A_6 + 32A_7), & Y_1 &= -2A_1 + 16A_2 - 4A_3 + 8A_4, \\ Y_2 &= -4A_2 + 2A_4 - 16A_5 + 8A_6 + 16A_7, & Z &= \frac{1}{840}(A_1 - 16A_2 + 2A_3 + 16A_4 - 32A_5 + 16A_6 - 32A_7), \end{aligned} \quad (2.6)$$

To derive (2.6) we have used pair exchange and cyclic identities for the Riemann tensor, and dropped terms containing Ricci tensors. Note that  $X + 6Y_1 - 24Y_2 = 0$ .

The relevant contributions to the supersymmetry transformation rules are presented in table 1. In table 2 we show the generic structure of the variations of the action that we have considered. These variations may generate

<sup>#3</sup>  $\psi$  indicates the gravitino,  $\psi_{(2)}$  indicates the gravitino curvature.

Table 1

The schematic form of the supersymmetry transformation rules considered in this paper. The symbol  $\psi$  represents the gravitino,  $\psi_{(2)}$  the gravitino curvature.

| # | Transformation   |
|---|--|
| 1 | $\delta\psi = \mathcal{D}(\omega)\epsilon$                                 |
| 2 | $\delta H = \bar{\epsilon}\psi_{(2)}, \quad \delta B = \bar{\epsilon}\psi$ |
| 3 | $\delta\omega = \bar{\epsilon}\psi_{(2)}$                                  |
| 4 | $\delta\psi_{(2)} = \epsilon R$  |
| 5 | $\delta e = \bar{\epsilon}\psi$  |

Table 2

The different structures in the variation of the action. The remainders indicates terms that may be left over after cancellation (see below) and can be shifted to a later stage of the calculations.

| #   | Variation  | Remainder           |
|-----|--|---------------------|
| (A) | $\bar{\epsilon}\psi_{(2)}R^2 \mathcal{D}R$       | (2.8)               |
| (B) | $\bar{\epsilon} \mathcal{D}\psi_{(2)} R^3$       | (2.7), (2.8)–(2.10) |
| (C) | $\bar{\epsilon}\psi R(\mathcal{D}R)^2$           | –                   |
| (D) | $\bar{\epsilon}\psi R^2 \mathcal{D}\mathcal{D}R$ | (2.11)              |
| (E) | $\bar{\epsilon}\psi R^4$                         | –                   |

Table 3

All contributions to the variations (A)–(E), given in table. The numbers in the table correspond to the supersymmetry transformation in table 1.

| Contribution                                 | (A) | (B) | (C) | (D) | (E) |
|--|-----|-----|-----|-----|-----|
| $R^4$  | 3   | –   | –   | –   | 5   |
| $R^2 \mathcal{D}RH$                          | 2   | –   | –   | –   | –   |
| $\bar{\psi}_{(2)} \mathcal{D}\psi_{(2)} R^2$ | 4   | 4   | –   | –   | –   |
| $\bar{\psi}_{(2)} \psi_{(2)} R \mathcal{D}R$ | 4   | –   | –   | –   | –   |
| $\bar{\psi} \psi_{(2)} R^3$                  | 1   | 1   | –   | –   | 4   |
| $\bar{\psi} \psi R^2 \mathcal{D}R$           | 1   | –   | 1   | 1   | –   |
| $BR^4$                                       | –   | –   | –   | –   | 2   |

a remainder that has to be taken into account in a subsequent cancellation. To give an example, the Bianchi identity for the gravitino curvature reads

$$\mathcal{D}_{[c} \psi_{ab]} = -\frac{1}{4} \Gamma^{ef} \psi_{[c} R_{ab]ef}. \quad (2.7)$$

Using this Bianchi identity we can simplify the variations of the type  $\bar{\epsilon} \mathcal{D}\psi_{(2)} R^3$ , at the expense of introducing an additional contribution to the cancellation of the  $\bar{\epsilon} \psi R^4$  variation. In this way we try to move, where possible, terms from the (A)–(D) cancellations to (E). There are, besides (2.7), a number of other identities which we use in this way. Some involve the equations of motion <sup>#4</sup>,  $\Psi^\mu$  and  $\Lambda$  of the gravitino  $\psi_\mu$  and of  $\lambda$ , respectively. They read:

$$\Gamma^b \psi_{ab} = \Psi_a + \frac{1}{4} \sqrt{2} \Gamma_a \Lambda - 2 \sqrt{2} \mathcal{D}_a \lambda, \quad (2.8)$$

$$\not{\mathcal{D}} \psi_{ab} = -2 \not{\mathcal{D}}_{[a} \Psi_{b]} + \frac{1}{2} \sqrt{2} \Gamma_{[a} \mathcal{D}_{b]} \Lambda - \frac{1}{4} \Gamma^c \Gamma^{ef} \psi_c R_{abef} - \frac{1}{2} \sqrt{2} \Gamma^{cd} \lambda R_{abcd} + \Gamma^c \psi_{[a} R_{b]c}, \quad (2.9)$$

$$\mathcal{D}^b \psi_{ab} = \not{\mathcal{D}} \Psi_a - \frac{1}{4} \sqrt{2} (\Gamma_a \not{\mathcal{D}} \Lambda - \mathcal{D}_a \Lambda) - \frac{1}{4} \Gamma^{ef} \psi^b R_{abef} + \frac{1}{2} R_{ab} (\Gamma^{bc} \psi_c - \psi^b) + \frac{1}{4} R \psi_a, \quad (2.10)$$

$$\mathcal{D}_{[\mu} \mathcal{D}_{\nu]} R_{abcd} = -R_{\mu\nu[a} \not{R}_{b]cd} - R_{\mu\nu[c} \not{R}_{ab]d}]. \quad (2.11)$$

Remaining contributions in the variation of the action which contain the equations of motion can be cancelled by modifying the transformation rules of the supergravity fields with terms proportional to  $\gamma$ . In table 3 the calculation is presented schematically. As explained above, the use of the relations (2.7)–(2.11) gives additional contributions to the variations of the form (E).

In practice we first considered the cancellations (C) and (D), because these restrict only the coefficients of the  $\bar{\psi} \psi R^2 \mathcal{D}R$  terms. Note that in principle a term cubic in the Riemann tensor, with two additional derivatives, could also contribute to this sector. However, all such terms vanish due to identities for the Riemann tensor, or contain Ricci tensors.

After performing and simplifying all variations, we solve a system of linear equations for the remaining coefficients. We find that two independent solutions remain. In the next section we will discuss their properties.

### 3. Results

Using the Noether procedure, as discussed in the previous section, we find that supersymmetry requires that the  $R^4$ -terms must occur in the following combination:

$$\mathcal{L} = aA_1 + (-16a+b)A_2 + 2aA_3 + (12a-2b)A_4 + (-32a+4b)A_5 + (16a-2b)A_6 + (-16a+2b)A_7. \quad (3.1)$$

<sup>#4</sup> These equations of motion are defined as  $\Psi^\mu \equiv (\partial/\partial\psi_\mu) \mathcal{L}_R$  and  $\Lambda \equiv (\partial/\partial\lambda) \mathcal{L}_R$ , where  $\mathcal{L}_R$  is the  $N=1$ ,  $d=10$  supergravity  $R$ -action, as given in ref. [13].

Expressed in terms of  $X$ ,  $Y_1$ ,  $Y_2$  and  $Z$  this reads

$$\mathcal{L} = cX + \frac{1}{8} \times 7! (a - \frac{1}{8}b)Z + [6c - (\frac{1}{8}a + \frac{3}{64}b)]Y_1 + [-24c + \frac{1}{2}(a - \frac{1}{8}b)]Y_2. \quad (3.2)$$

Here the coefficient  $c$  is arbitrary and reflects the dependence of  $X$ ,  $Y_1$  and  $Y_2$  discussed in section 2. We will choose  $c = \frac{1}{48}(a - \frac{1}{8}b)$ , and associate one independent solution with the choice  $b=0$ , the other with  $b=8a$ . The arbitrary scale  $a$  is set equal to one.

The bosonic part of the first invariant reads

$$\begin{aligned} I_1 = & e(R_{abef}R_{abef}R_{cdgh}R_{cdgh} - 16R_{acef}R_{bcef}R_{adgh}R_{bdgh} + 2R_{abef}R_{cdef}R_{abgh}R_{cdgh} + 12R_{abef}R_{cdef}R_{acgh}R_{bdgh} \\ & - 32R_{abef}R_{cdef}R_{agch}R_{bgdh} + 16R_{acef}R_{bcef}R_{agbh}R_{cgdh} - 16R_{acef}R_{bcef}R_{agch}R_{bgdh}) \\ & + \frac{1}{8}i\sqrt{2}\epsilon^{\mu_1\dots\mu_{10}}B_{\mu_1\mu_2}R_{\mu_3\mu_4}{}^{ab}R_{\mu_5\mu_6}{}^{ab}R_{\mu_7\mu_8}{}^{cd}R_{\mu_9\mu_{10}}{}^{cd} - \frac{1}{2}i\sqrt{2}\epsilon^{\mu_1\dots\mu_{10}}B_{\mu_1\mu_2}R_{\mu_3\mu_4}{}^{ab}R_{\mu_5\mu_6}{}^{ac}R_{\mu_7\mu_8}{}^{bd}R_{\mu_9\mu_{10}}{}^{cd}. \end{aligned} \quad (3.3)$$

The  $R^4$ -terms in (3.3) correspond to  $\frac{1}{48}[X + (6 \times 7!)Z]$ . Note that in (3.3) no terms linear in  $H$  are present [in our calculation the argument of the Riemann tensor is always the spin-connection  $\omega(e, \psi)$ ]. In ref. [4] it was found that in the string effective action the Riemann tensor should depend on the spin-connection  $\Omega_-$  [see (2.2)]. However, when  $X$  and  $Z$  are written in terms of  $\Omega_-$ , and one then expands in  $H$ , terms linear in  $H$  cancel. Thus the effect of torsion appears only in the terms quadratic in  $H$ , which we do not consider here. The presence of  $K_1$  and  $K_2$  in (3.3), corresponding to a five-point amplitude in a string calculation, was anticipated in ref. [10].

The bosonic part of the second invariant is given by

$$\begin{aligned} I_2 = & e[R_{abef}R_{abef}R_{cdgh}R_{cdgh} - 8R_{acef}R_{bcef}R_{adgh}R_{bdgh} + 2R_{abef}R_{cdef}R_{abgh}R_{cdgh} - 4R_{abef}R_{cdef}R_{acgh}R_{bdgh} \\ & + 96\sqrt{2}H^{abc}(-R_{abem}R_{ghfm}\mathcal{D}_eR_{efgh} + R_{abem}R_{ghfm}\mathcal{D}_eR_{cfgh})] \\ & + \frac{1}{8}i\sqrt{2}\epsilon^{\mu_1\dots\mu_{10}}B_{\mu_1\mu_2}R_{\mu_3\mu_4}{}^{ab}R_{\mu_5\mu_6}{}^{ab}R_{\mu_7\mu_8}{}^{cd}R_{\mu_9\mu_{10}}{}^{cd}. \end{aligned} \quad (3.4)$$

The  $R^4$ -terms in (3.4) are  $-\frac{1}{2}Y_1$  (1.3). In this case we do obtain contributions linear in  $H$ . Using the pair exchange of the Riemann tensor, all  $R^4$ -terms can be rewritten in terms of  $Y_{\mu\nu\lambda\rho} \equiv R_{\mu\nu}{}^{ab}(\omega)R_{\lambda\rho}{}^{ab}(\omega)$  and its contractions. Note that  $Y_{\mu\nu\lambda\rho}$  has an analogue for an arbitrary Yang–Mills group:  $\text{tr } F_{\mu\nu}F_{\lambda\rho}$ . The analogy between  $\Omega_-$  and the gauge field  $A_\mu$  of the super-Yang–Mills multiplet [see (2.3)] suggests that the terms linear in  $H$  in (3.4) and the  $R^4$ -terms should combine if we introduce  $\Omega_-$  as the spin-connection. Indeed, the terms linear in  $H$  can both be absorbed in this way.

In this short communication we will refrain from giving the terms in the action which depend on the fermions. One surprise (for us) in this fermionic sector is that all terms of the type  $\bar{\psi}\psi R^2 \mathcal{D}R$  have a vanishing coefficient. The fermionic terms corresponding to the solution (3.4) can all be written in terms of  $Y_{\mu\nu\lambda\rho}$  and contractions between  $R_{\mu\nu}{}^{ab}$  and  $\psi_{ab}$ , which can also be generalized to arbitrary Yang–Mills groups.

The remaining contributions to the variations proportional to the gravitino equation of motion are, for the invariant  $I_1$  (3.3),

$$\begin{aligned} & (R_{abcd}R_{ajef}R_{bkgh} - \frac{1}{4}R_{abcd}R_{abef}R_{jkgh})\bar{\epsilon}\Gamma_{cdefgh}\not{D}\psi_{jk} + (2R_{abde}R_{abci}R_{fgcj} + 12R_{acde}R_{afbi}R_{bgcj})\bar{\epsilon}\Gamma_{defg}\not{D}\psi_{ij} \\ & + (-8R_{acbd}R_{aebh}R_{cfdi} + 4R_{abcd}R_{abce}R_{dfhi} + 4R_{bcad}R_{efah}R_{bcdi} + 20R_{bcae}R_{adfh}R_{bcdi} + 2R_{adef}R_{bcah}R_{bcdi} \\ & - 16R_{abde}R_{cfah}R_{bcdi} + 24R_{abde}R_{afch}R_{bcdi} - \frac{1}{2}R_{abcd}R_{abcd}R_{efhi} + 2R_{abce}R_{abdf}R_{cdhi} - R_{abcd}R_{abef}R_{cdhi} \\ & + 8R_{acbe}R_{adbf}R_{cdhi})\bar{\epsilon}\Gamma_{ef}\not{D}\psi_{hi} \\ & - 2R_{abde}R_{acfg}R_{bhci}\bar{\epsilon}\Gamma_{defgh}\not{D}\psi_{ij} \\ & + (8R_{abce}R_{abdf}R_{cdgh} - 4R_{acbe}R_{adbf}R_{cdgh} - 8R_{abef}R_{acdgh}R_{bcdh} + 8R_{adef}R_{bcag}R_{bcdh})\bar{\epsilon}\Gamma_{efg}\not{D}\psi_{hi} \\ & + (32R_{acbe}R_{adbf}R_{cdeg} - 20R_{abcd}R_{abef}R_{cdeg} + 20R_{abcd}R_{abce}R_{dfeg})\bar{\epsilon}\Gamma_f\not{D}\psi_{gh}. \end{aligned} \quad (3.5)$$

Using (2.9) and (2.10) this can be expressed in terms of derivatives of  $\Psi^\mu$ , after which the required additional variations of the gravitino can be read off.

For the invariant  $I_2$  (3.4) the remaining fermionic equations of motion are

$$-\frac{1}{4}R_{cdab}R_{efab}R_{ghjk}\bar{\epsilon}\Gamma_{cdefgh}\Psi_{jk} + (2R_{ceab}R_{dfab}R_{cdhi} - \frac{1}{2}R_{cdab}R_{cdab}R_{efhi} - R_{cdab}R_{efab}R_{cdhi} \\ + 4R_{cdab}R_{ceab}R_{dfhi})\bar{\epsilon}\Gamma_{ef}\Psi_{hi}. \quad (3.6)$$

The bosonic equations of motion, which are required to determine the additional transformation rules of the zehnbein, will be presented elsewhere.

Let us now come back to (3.4). All contributions to (3.4) can be generalized to the  $d=10$  Yang–Mills multiplet, if we replace  $\Omega_-$  by  $A$ ,  $R(\Omega_-)$  by  $F(A)$ , and  $\psi_{(2)}$  by  $\chi$ , where  $\chi$  is the fermionic partner of the Yang–Mills gauge field. Invariance of (3.4) requires modifications of the transformation rules, which arise from remaining equations of motions in the variation of the action. In the case of (3.4), all these equations of motion correspond, after the above substitution, to the  $A$ - and  $\chi$ -equations of motion that follow from the  $F^2$ -action. For the fermionic equation of motion this can be seen from (3.6). Therefore (iterative) invariance requires the presence of the  $F^2$ -action [15]. As a byproduct of our analysis of  $R^4$ -actions we therefore find also the following  $\text{tr } F^2 + \gamma(\text{tr } F^2)^2$ -invariant coupled to supergravity:

$$\mathcal{L}_{\text{YM}} = \mathcal{L}_R + \mathcal{L}_{F^2} + \gamma e \{ -\frac{1}{2}t^{\mu_1 \dots \mu_8} \text{tr } F_{\mu_1 \mu_2} F_{\mu_3 \mu_4} \text{tr } F_{\mu_5 \mu_6} F_{\mu_7 \mu_8} \\ + \frac{1}{8}i\sqrt{2} e^{-1} \epsilon^{\mu_1 \dots \mu_{10}} B_{\mu_1 \mu_2} \text{tr } F_{\mu_3 \mu_4} F_{\mu_5 \mu_6} \text{tr } F_{\mu_7 \mu_8} F_{\mu_9 \mu_{10}} + 4\bar{X}^{\mu\nu} \Gamma^\lambda \text{tr}(\chi \mathcal{D}_\lambda F_{\mu\nu}) - 2 \text{tr}(\bar{\chi} \Gamma^{\mu\nu\lambda} \chi) \text{tr}(F_{\rho\lambda} \mathcal{D}^\rho F_{\mu\nu}) \\ + 4\bar{X}^{\mu\nu} \Gamma_{\mu\lambda\rho} \text{tr}(\chi \mathcal{D}_\nu F^{\lambda\rho}) - 8 \text{tr}(\bar{\chi} \Gamma_\mu \mathcal{D}_\nu \chi) \text{tr } F^{\mu\lambda} F^\nu{}_\lambda - 16\bar{X}^{\mu\nu} \Gamma_\mu \text{tr}[(\mathcal{D}^\rho \chi) F_{\rho\nu}] + \text{Noether terms} \}, \quad (3.7)$$

where  $X_{\mu\nu} \equiv \text{tr } F_{\mu\nu} \chi$ . Modifications of the Yang–Mills transformation rules are of  $\mathcal{O}(\gamma)$ . In the abelian case (3.7) is the quartic contribution to the Born–Infeld action [8] coupled to supergravity, and agrees in the flat limit with the globally supersymmetric Born–Infeld action presented in ref. [16]. In the Yang–Mills case the structure of (3.7) differs in the flat limit from the result of ref. [16], since in ref. [16] only the symmetric Yang–Mills trace (i.e.,  $\text{tr } F^4$ ) is considered.

In the present calculation we have not yet taken into account the  $\lambda$  and  $\phi$  dependence of the action. The form in which  $\phi$  appears can be anticipated from, e.g., Gross and Sloan [4], where it is shown that the action contains a multiplicative  $\phi^{-3}$  factor (in our notation).

#### 4. Discussion

In this paper, we have found that two supersymmetric invariants of the type  $R + \gamma R^4$  exist. As a by-product, we have also obtained the leading terms of a locally supersymmetric  $\text{tr } F^2 + \gamma(\text{tr } F^2)^2$ -invariant.

Amplitude calculations for the heterotic string tell us that the actions (3.3) and (3.4) are indeed part of the effective action for the heterotic string. At the one-loop level [17], other terms are obtained as well. For instance, in ref. [18] an additional  $F^4$ -term of the form  $t^{\mu_1 \dots \mu_8} \text{tr } F_{\mu_1 \mu_2} \dots F_{\mu_7 \mu_8}$  is found. Such a term would be the generalization of our  $Y_2$  (1.3) to an arbitrary Yang–Mills group, in the same way that (3.7) is the generalization of (3.4). However, from (3.2) we see that it is not possible to obtain an invariant  $R + \gamma \text{tr } F^4$ , not even if we allow the presence of an  $F^2$ -term. This is not in disagreement with refs. [17,18], because that result included couplings of the type  $\text{tr } R^2 \text{tr } F^2$ , which were not considered here.

In a recent paper by Duff and Lu [19] it was argued that the coupling of the heterotic five-brane [20]  $\sigma$ -model to background supergravity fields implies the existence of a  $\text{tr } R^4 + \text{tr } R^2(\text{tr } F^2 + \text{tr } R^2) + \text{tr } F^4$ -action, which is similar to the result of refs. [17,18].

From the comparison of our result (3.2) with the results of refs. [17,18] and ref. [19] we conclude that the  $\text{tr } R^2 \text{tr } F^2$ -coupling is essential in the supersymmetrization of a  $\text{tr } F^4$ -term.

We can introduce certain  $\text{tr } R^2 \text{ tr } F^2$ -terms in the following way. Since (3.7) is an invariant for an arbitrary Yang–Mills group  $G$ , we can also use it for the group  $G \otimes \text{SO}(9, 1)$ . This leads to an invariant which contains  $(\text{tr } R^2 + \text{tr } F^2)^2$ . However, it requires the presence of an  $R^2$ -action, besides  $F^2$ . This is precisely the situation considered in ref. [13]. The analysis of ref. [13] differs from the one done here, because the presence of the  $R^2$ -action causes modifications to the transformation rules (2.1) and in particular (2.3). This leads to a different quartic action.

An interesting feature of our work is the appearance of the  $B \wedge R \wedge R \wedge R \wedge R$  terms. They are related to Chern–Simons terms. The usual Lorentz Chern–Simons term appears as a modification to the field strength  $H$  of the gauge field  $B$ , schematically, this reads  $H \rightarrow \partial B + \text{tr } \omega \wedge \partial \omega + \omega \wedge \omega \wedge \omega$ , along with the Yang–Mills Chern–Simons term [15]. There is an alternative version of  $d=10$ ,  $N=1$  supergravity in which a six-index tensor gauge field  $A_{(6)}$  is used instead of  $B_{(2)}$  [27]. In that version Chern–Simons terms are absent, but are replaced by an interaction term of the form  $A_{(6)} \wedge R \wedge R$  in the action. The two versions are related by a duality transformation, which can be extended to the quartic action required for the supersymmetrization of this interaction term [22].

By a similar duality transformation, the terms  $B \wedge R \wedge R \wedge R \wedge R$  will give rise to Chern–Simons terms of the type

$$H_{(7)} \rightarrow \partial A_{(6)} + \omega \wedge \partial \omega \wedge \partial \omega \wedge \partial \omega + \dots \quad (4.1)$$

in the seven-index field strength of  $A_{(6)}$  in the six-index version of  $d=10$  supergravity. Such terms are indeed required in the anomaly cancellations in the six-index version [23].

The terms (2.4) also appear as counterterms in the anomaly cancellation in the usual two-index formulation of supergravity [9]. Thus we find the supersymmetrization of (some of) the counterterms required for the anomaly free theory. It is interesting to note, that the (quantum) counterterms in one version are related, by a duality transformation, to a (classical) Chern–Simons term in the other version. This quantum aspect of duality transformations is also discussed in ref. [19] in the context of string/five-brane duality.

Details of our Noether construction, the complete actions including fermionic terms, as well as the completion of our result to include  $\phi$  and  $\lambda$  contributions, will be published elsewhere. Our construction does not rigorously prove the existence of the invariants. This is due to the restrictions required to keep the calculation (done by a program for algebraic manipulations) within reasonable limits, but also to the iterative character of the invariant itself. However, because we find agreement with known bosonic actions from string amplitude calculations, we are confident that indeed the invariants given in section 3 can be extended beyond the practical limitations imposed here.

The results of this paper complete the supersymmetrization of the quartic string effective action corresponding to tree-level string amplitudes. This opens the possibility to reconsider the string corrections to the compactification procedure of ref. [11]. To obtain the supersymmetric completion of the terms which come from one-loop string calculations (these terms also play a role in the discussion of five-brane/string duality) requires further work.

## Acknowledgement

We thank Eric Bergshoeff for a number of useful discussions. This work is financially supported by the Stichting voor Fundamenteel Onderzoek der Materie (FOM). One of us (A.W.) gratefully acknowledges the financial support of the Deutsche Forschungs Gemeinschaft (DFG) under contract nr. Wi 1033/1-1.



## References

- [1] D.J. Gross and E. Witten, Nucl. Phys. B 277 (1986) 1.
- [2] M.T. Grisaru, A.E.M. van de Ven and D. Zanon, Nucl. Phys. B 277 (1986) 389.
- [3] J.H. Schwarz, Phys. Rep. 89 (1982) 223.
- [4] Y. Kikuchi, C. Marzban and Y.J. Ng, Phys. Lett. B 176 (1986) 57;  
Y. Cai and C.A. Núñez, Nucl. Phys. B 287 (1987) 279;  
D.J. Gross and J.H. Sloan, Nucl. Phys. B 291 (1987) 41.
- [5] B. Zumino, Phys. Rep. 137 (1986) 109.
- [6] H. Nishino and S.J. Gates Jr., Nucl. Phys. B 282 (1987) 1.
- [7] R. Myers, Nucl. Phys. B 289 (1987) 701.
- [8] M. Born and L. Infeld, Proc. R. Soc. A 144 (1934) 425.
- [9] M.B. Green and J.H. Schwarz, Phys. Lett. B 149 (1984) 117.
- [10] W. Lerche, B.E.W. Nilsson and A.N. Schellekens, Nucl. Phys. B 289 (1987) 609;  
J. Ellis and L. Mizrachi, Nucl. Phys. B 302 (1988) 65.
- [11] P. Candelas, G.T. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B 258 (1985) 46.
- [12] B.E.W. Nilsson and A.K. Tollstén, Phys. Lett. B 181 (1986) 63;  
R. Kallosh, Phys. Scr. T15 (1987) 118;  
K. Lechner and P. Pasti, Mod. Phys. Lett. A 4 (1989) 1721.
- [13] E.A. Bergshoeff and M. de Roo, Nucl. Phys. B 328 (1989) 439.
- [14] A.H. Chamseddine, Nucl. Phys. B 185 (1981) 403;  
E. Bergshoeff, M. de Roo, B. de Wit and P. van Nieuwenhuizen, Nucl. Phys. B 195 (1982) 97.
- [15] G.F. Chapline and N.S. Manton, Phys. Lett. B 120 (1983) 105.
- [16] E. Bergshoeff, M. Rakowski and E. Sezgin, Phys. Lett. B 185 (1987) 371.
- [17] N. Sakai and Y. Tanii, Nucl. Phys. B 287 (1987) 457.
- [18] J. Ellis, P. Jetzer and L. Mizrachi, Nucl. Phys. B 303 (1988) 1;  
M. Abe, H. Kubota and N. Sakai, Nucl. Phys. B 306 (1988) 405.
- [19] M.J. Duff and J.X. Lu, Nucl. Phys. B 357 (1991) 543.
- [20] A. Ströminger, Nucl. Phys. B 343 (1990) 167.
- [21] A.H. Chamseddine, Phys. Rev. D 24 (1981) 3065.
- [22] E. Bergshoeff and M. de Roo, Phys. Lett. B 249 (1990) 27.
- [23] S.J. Gates Jr. and H. Nishino, Phys. Lett. B 157 (1985) 157.